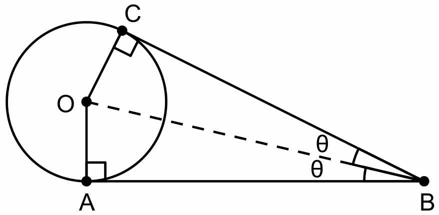
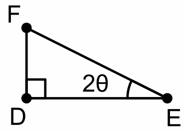
TRIGONOMETRIA – PARTE 3

QUESTÃO 01

Considere a figura, em que $\overline{AO} = \overline{OC} = r$ é a medida do raio da esfera e $A\hat{B}C = 2\theta$.





Sendo $\overline{AB} = 10\text{ m}$, temos

$$\operatorname{tg} A\hat{B}O = \frac{\overline{AO}}{\overline{AB}} \Leftrightarrow \operatorname{tg}\theta = \frac{r}{10}.$$

Por outro lado, como $BC \parallel EF$, $\overline{DF} = 1\text{m}$ e $\overline{DE} = 2\text{ m}$, vem

$$\operatorname{tg} D\hat{E}F = \frac{\overline{DF}}{\overline{DE}} \Leftrightarrow \operatorname{tg} 2\theta = \frac{1}{2}$$

$$\Leftrightarrow \frac{2\operatorname{tg}\theta}{1-\operatorname{tg}^2\theta} = \frac{1}{2}$$

$$\Leftrightarrow \frac{2\cdot\frac{r}{10}}{1-\left(\frac{r}{10}\right)^2} = \frac{1}{2}$$

$$\Leftrightarrow r^2 + 40r - 100 = 0$$

$$\Rightarrow r = (10\sqrt{5} - 20)\text{ m}.$$

Letra B

QUESTÃO 02

$$\cos x = 1 - \left(-\frac{1}{3}\right)^2 \Rightarrow \cos^2 x = \frac{8}{9} \Rightarrow \cos x = \pm\frac{2\sqrt{2}}{3}$$

Como $\pi < x < \frac{3\pi}{2}$, temos: $\cos x = -\frac{2\sqrt{2}}{3}$

Portanto:

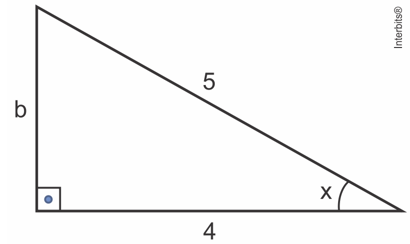
$$\operatorname{sen} 2x = 2\operatorname{sen} x \cdot \cos x$$

$$\operatorname{sen} 2x = 2\cdot\left(-\frac{1}{3}\right)\cdot\left(-\frac{2\sqrt{2}}{3}\right) = \frac{4\cdot\sqrt{2}}{9}$$

Letra E

QUESTÃO 03

Se $\cos x = \frac{4}{5}$ e $x \in \left[0, \frac{\pi}{2}\right]$, podemos considerar um triângulo retângulo com um dos ângulos agudos medindo x o cateto adjacente a ele medindo 4 e a hipotenusa medindo



Calculando a medida do cateto b através do Teorema de Pitágoras, podemos escrever:

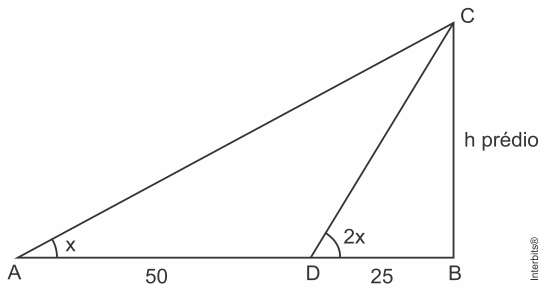
$b^2 + 4^2 = 5^2 \Rightarrow b = 3.$

Concluímos então que $\operatorname{tg} x = \frac{3}{4}$ e que:

$$\operatorname{tg}(2x) = \frac{2\cdot\operatorname{tg} x}{1-\operatorname{tg}^2 x} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2}\cdot\frac{16}{7} = \frac{24}{7}.$$

Letra C

QUESTÃO 04



Sendo h a altura do prédio, pode-se escrever:

$$\operatorname{tg}(x) = \frac{h}{75}$$

$$\operatorname{tg}(2x) = \frac{h}{25}$$

$$\operatorname{tg}(2x) = \frac{2\cdot\operatorname{tg}(x)}{1-(\operatorname{tg}(x))^2}$$

$$\frac{h}{25} = \frac{2\cdot\frac{h}{75}}{1-\left(\frac{h}{75}\right)^2} \to \frac{1}{25} = \frac{2}{75}\cdot\frac{75^2}{75^2 - h^2} \to h^2 = 1875 \to h = 25\sqrt{3}$$

Letra D

QUESTÃO 05

Desenvolvendo os quadrados, obtemos

$$f(x) = (\operatorname{sen} x + \cos x)^2 + (\operatorname{sen} x - \cos x)^2$$

$$= \underbrace{\operatorname{sen}^2 x + \cos^2 x}_{1} + 2\operatorname{sen} x\cos x + \underbrace{\operatorname{sen}^2 x + \cos^2 x}_{1} - 2\operatorname{sen} x\cos x$$

$$= 2.$$

Portanto, como f é constante, segue que a alternativa B é a que apresenta um possível gráfico de f.

Letra B

QUESTÃO 06

y = 1

Letra C

QUESTÃO 07

f(x) = 8.(2.senx.cosx) = 8.sen(2.x)

Letra D

QUESTÃO 08

$\text{tgx} = 2 \rightarrow \frac{\text{senx}}{\text{cosx}} = 2 \rightarrow \text{senx} = 2.\text{cosx}$

$\text{sen}^2x + \cos^2 x = 1 \rightarrow \text{cosx} = \frac{1}{\sqrt{5}} \text{ e senx} = \frac{2}{\sqrt{5}}$

$\cos(2x) = \cos^2 x - \text{sen}^2 x = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$

$\text{sen}(2.x) = 2.\text{senx}.\text{cosx} = \frac{4}{5}$

$\frac{\cos(2x)}{1+\text{sen}(2.x)} = \frac{\frac{-3}{5}}{\frac{9}{5}} = \frac{-3}{9} = \frac{-1}{3}$

Letra B

QUESTÃO 09

$\cos\vartheta = \frac{15}{20} = \frac{3}{4}$

$\cos(2.\vartheta) = 2.\cos^2\vartheta - 1$

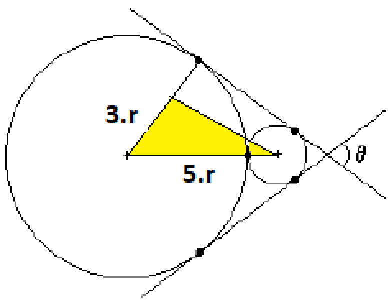
$\cos(2.\vartheta) = 2.(\frac{3}{4})^2 - 1 = \frac{9}{8} - 1 = \frac{1}{8}$

Letra E

QUESTÃO 10

No triângulo hachurado a medida da hipotenusa é 5.r = 4.r + r.

Um dos catetos mede 3.r = 4.r – r e por Pitágoras o outro cateto mede 3.r.



$\textbf{sen}\left(\frac{\vartheta}{2}\right) = \frac{3.r}{5.r} = \frac{3}{5} \textbf{ e cos}\left(\frac{\vartheta}{2}\right) = \frac{4.r}{5.r} = \frac{4}{5}$

$\cos(\vartheta) = 2.\cos^2\left(\frac{\vartheta}{2}\right) - 1 = 2.(\frac{4}{5})^2 - 1 = \frac{7}{25}$

Letra E

QUESTÃO 11

$A = 2.(\text{sena}.\text{cosb} + \text{senb}.\text{cosa})$

$A = 2.\text{sen}(a+b) = 2.\text{sen}\left(\frac{\pi}{6}\right) = 2.\frac{1}{2} = 1$

Letra D

QUESTÃO 12

$\text{tg}\alpha = \frac{6}{x} \text{ e } \text{tg}(2.\alpha) = \frac{24}{x}$

$\text{tg}(2.\alpha) = \frac{2.\text{tg}\alpha}{1-\text{tg}^2\alpha}$

$\frac{24}{x} = \frac{\frac{12}{x}}{1-\frac{36}{x^2}} \rightarrow 2 = \frac{1}{1-\frac{36}{x^2}} \rightarrow x = 8{,}5 \text{ m}$

Letra A

QUESTÃO 13

$\text{sen}(2.\theta) = 2.\text{sen}\theta.\cos\theta$

$2.p = 2.3.p.\cos\theta \rightarrow \cos\theta = \frac{1}{3}$

$\text{sen}^2\theta + \cos^2\theta = 1 \rightarrow \text{sen}\theta = \frac{2.\sqrt{2}}{3}$

$3.p = \frac{2.\sqrt{2}}{3} \rightarrow p = \frac{2.\sqrt{2}}{9}$

Letra D

QUESTÃO 14

Tomando CD = x

$\text{tg}(2.\theta) = \frac{3}{\sqrt{3}} = \sqrt{3} \rightarrow 2.\theta = 60^0 \text{e } \theta = 30^0$

$\text{tg}(30^0) = \frac{\sqrt{3}}{3} = \frac{3}{AC} \rightarrow AC = 3.\sqrt{3}$

$CD = AC - AD = 3.\sqrt{3} - \sqrt{3} = 2.\sqrt{3}$

$\text{Área} = \frac{CD.AB}{2} = 3.\sqrt{3}$

Letra D

QUESTÃO 15

$\text{sen}(2.\pi - x) + \text{sen}(3.\pi + x) = -\text{senx} - \text{senx}$

$\text{sen}(2.\pi - x) + \text{sen}(3.\pi + x) = -2.\text{senx}$

Letra D

QUESTÃO 16

$\text{sen}(60^0 + 45^0) = \text{sen}60^0.\cos45^0 + \cos60^0.\text{sen}45^0$

$\text{sen}(105^0) = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos(60^0 - 45^0) = \cos60^0.\cos45^0 + \text{sen}60^0.\text{sen}45^0$

$\text{sen}(15^0) = \frac{\sqrt{6}+\sqrt{2}}{4}$

Letra A

QUESTÃO 17

$\begin{cases} x + y = 90^0 \\ x - y = 60^0 \end{cases} \rightarrow x = 75^0 \text{e } y = 15^0$

$\text{senx} + \text{seny} = 2.\text{sen}\left(\frac{x+y}{2}\right).\cos(\frac{x-y}{2})$

$\text{sen}75^0 + \text{sen}15^0 = 2.\text{sen}45^0.\cos30^0$

$\text{sen}75^0 + \text{sen}15^0 = 2.\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

Letra C

QUESTÃO 18

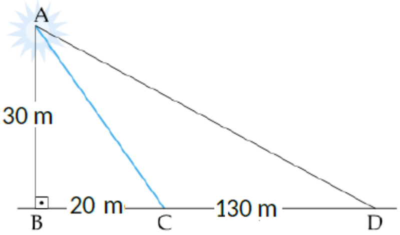
$tg^2\vartheta = \frac{9}{16} \rightarrow tg\vartheta = \frac{3}{4} \rightarrow sen\vartheta = \frac{3}{5} \rightarrow cos\vartheta = \frac{3}{5}$

$sen(2.\vartheta) = 2.sen\vartheta.cos\vartheta = 2.\frac{3}{5}.\frac{4}{5} = 0{,}96$

$\frac{sen\theta}{sen(2.\theta)} = \frac{0{,}60}{0{,}96} = 0{,}625 = 1 - 0{,}375$

Letra D

QUESTÃO 19



$tg(BAC) = \frac{2}{3}$ e $tg(BAD) = 5$

$tg(CAD) = tg(BAD - BAC)$

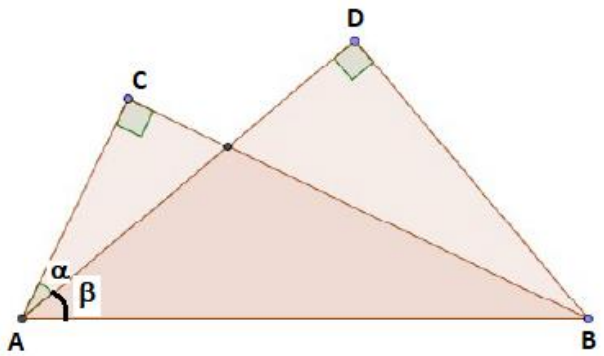
$tg(CAD) = \frac{5-\frac{2}{3}}{1+5.\frac{2}{3}} = \frac{\frac{13}{3}}{\frac{13}{3}} = 1$

Logo CAD = 45º

Letra B

QUESTÃO 20

Como AD = BD, então β = 45º.



AC = 1 e BC = 7, temos que $tg(\alpha + \beta) = 7$.

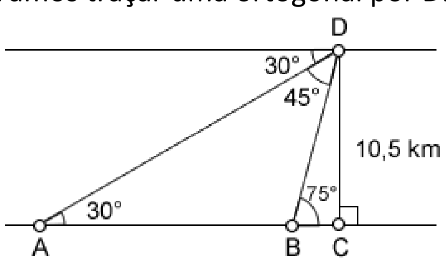
$tg(\alpha + 45^0) = \frac{tg\alpha + tg45^0}{1 - tg\alpha.tg45^0}$

$7 = \frac{tg\alpha + 1}{1 - 1.tg\alpha} \rightarrow tg\alpha = \frac{3}{4} \rightarrow sen\alpha = \frac{3}{5}$

Letra C

QUESTÃO 21

Vamos traçar uma ortogonal por D.



No triângulo no BCD:

$\tan 75^\circ = \frac{10,5}{BC} \Rightarrow \tan(30^\circ + 45^\circ) = \frac{10,5}{BC}$

$\frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \cdot \tan 45^\circ} = \frac{10,5}{BC} \Rightarrow$

$\frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}} = \frac{10,5}{BC} \Rightarrow BC = 10,5 \cdot (2 - \sqrt{3})$

No triângulo no ACD:

$\tan 30^\circ = \frac{10,5}{AC} \Rightarrow \frac{\sqrt{3}}{3} = \frac{10,5}{AC} \Rightarrow AC = 10,5\sqrt{3}$

$AB = AC - BC = 10,5\sqrt{3} - 10,5 \cdot (2 - \sqrt{3}) = 21 \cdot (\sqrt{3} - 1)$

Letra A

QUESTÃO 22

Elevando-se as duas expressões ao quadrado:

$$\begin{cases} cos^2a + 2.cosa.cosb + cos^2b = \frac{3}{2} \\ sen^2a + 2.sena.senb + sen^2b = \frac{1}{2} \end{cases}$$

Adicionando membro a membro:

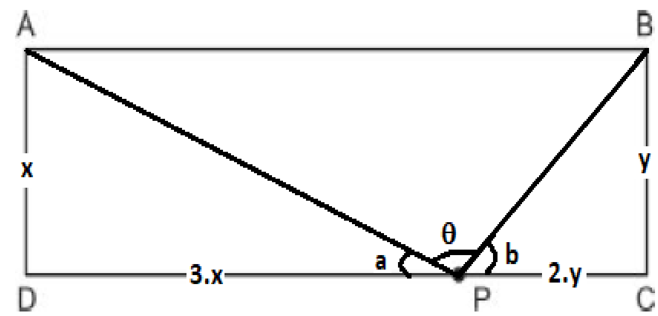
$1 + 2.(cosa.cosb + sena.senb) + 1 = 2$

$2.(cosa.cosb + sena.senb) = 0$

$cos(a - b) = 0$

Letra A

QUESTÃO 23



$tga = \frac{x}{3.x} = \frac{1}{3}$ e $tgb = \frac{y}{2.y} = \frac{1}{2}$

$tg(a + b) = \frac{tga + tgb}{1 - tga.tgb} = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}.\frac{1}{2}} = 1$

$a + b = 45^0 \rightarrow \theta = 135^0$

Letra A

QUESTÃO 24

$* \ A = cos^2a + 2.cosa.cosb + cos^2b + sen^2a + 2.sena.senb + sen^2b$

* A = 1 + 2.cos(a - b) + 1 = 1 + 2.cos60º + 1 = 3

Letra A

QUESTÃO 25

$\text{sen}(2.x) = 2.\text{senx}.\text{cosx}$

$\text{senx} = 2.\text{senx}.\text{cosx} \rightarrow \text{cosx} = \frac{1}{2}$

$x = 60^o \text{ou } x = 300^0$

Logo, a soma vale 360º.

Letra B

QUESTÃO 26

Para o triângulo ACD:

$\text{tg}\alpha = \frac{CD}{AC} \rightarrow \frac{CD}{17} = \frac{7}{17} \rightarrow CD = 7$

Para o triângulo BCE:

$\beta = 180 - (180 - 45 - \alpha)$

$\beta = 45 + \alpha$

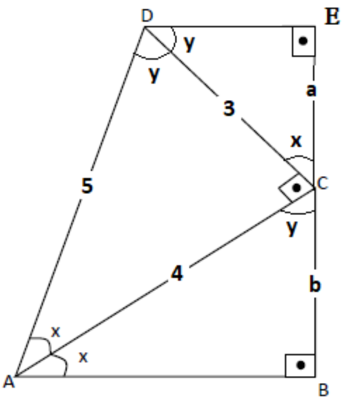
$\text{tg}\beta = \text{tg}(45^o + \alpha)$

$\text{tg}\beta = \frac{\text{tg}(45^o)+\text{tg}\alpha}{1-\text{tg}(45^o).\text{tg}\alpha} = \frac{1+\frac{7}{17}}{1-1.\frac{7}{17}} = \frac{\frac{24}{17}}{\frac{10}{17}} = 2.4$

$\text{tg}\beta = \frac{CE}{BC} \rightarrow \frac{CE}{5} = 2{,}4 \rightarrow CE = 12$

Letra A

QUESTÃO 27



Usando semelhança de triângulos:

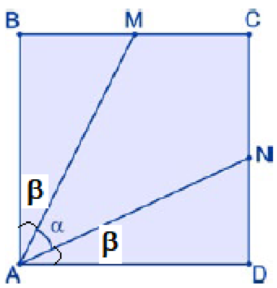
$\frac{a}{3} = \frac{4}{5} \rightarrow a = 2{,}4$

$\frac{b}{4} = \frac{3}{5} \rightarrow b = 2{,}4$

a + b = 4,8

Letra B

QUESTÃO 28



$\text{sen}\beta = \frac{1}{\sqrt{5}} \text{ e } \cos\beta = \frac{2}{\sqrt{5}}$

$\text{sen}(2.\beta) = 2.\text{sen}\beta.\cos\beta = \frac{4}{5}$

$\cos(2.\beta) = \cos^2\beta - \text{sen}^2\beta = \frac{3}{5}$

$\alpha + 2.\beta = 90^o \rightarrow \text{sen}(\alpha + 2.\beta) = \text{sen}90^o = 1$

$\text{sen}\alpha.\cos(2.\beta) + \cos\alpha.\text{sen}(2.\beta) = 1$

$\text{sen}\alpha.\frac{3}{5} + \cos\alpha.\frac{4}{5} = 1$

$3.\text{sen}\alpha + 4.\cos\alpha = 5$

Isolando cosα

$\cos\alpha = \frac{5-3.\text{sen}\alpha}{4}$

Combinando com:

$\text{sen}^2\alpha + \cos^2\alpha = 1$

$25.\text{sen}^2\alpha - 30.\text{sen}\alpha + 9 = 0$

$\text{sen}\alpha = \frac{3}{5}$

Letra A

QUESTÃO 29

$\text{sen}(2x) = \frac{h}{160} \text{ e } \text{sen}(4x) = \frac{h}{100}$

$\text{sen}(4x) = 2.\text{sex}(2x).\cos(2x)$

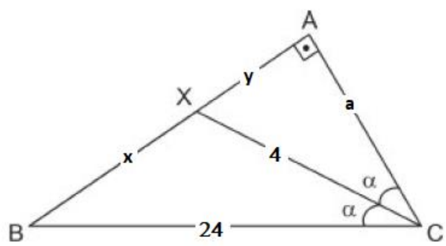
$\frac{h}{100} = 2.\frac{h}{160}.\cos(2x) \rightarrow \cos(2x) = \frac{4}{5}$

$\cos(2x) = \frac{4}{5} \rightarrow \text{sen}(2x) = \frac{3}{5}$

$\text{sen}(2x) = \frac{3}{5} = \frac{h}{160} \rightarrow h = 96\text{ m}$

Letra C

QUESTÃO 30



$a^2 = 16 - y^2$

$\frac{y}{a} = \frac{x}{24} \rightarrow y = \frac{a.x}{24}$

$a^2 = 576 + (x + y)^2$

a = 3 cm

Letra B