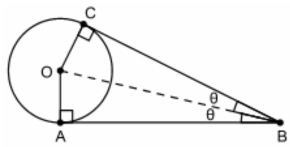
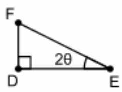
QUESTÃO 01

Considere a figura, em que $\overline{AO} = \overline{OC} = r$ é a medida do raio da esfera e $A\hat{B}C = 2\theta$.





Sendo $\overline{AB} = 10$ m, temos $\operatorname{tg} A\hat{B}O = \frac{\overline{AO}}{\overline{AB}} \Leftrightarrow \operatorname{tg}\theta = \frac{r}{10}$.

Por outro lado, como BC ∥ EF, $\overline{DF} = 1$ m e $\overline{DE} = 2$ m, vem

$$\operatorname{tg} D\hat{E}F = \frac{\overline{DF}}{\overline{DE}} \Leftrightarrow \operatorname{tg} 2\theta = \frac{1}{2} \Leftrightarrow \frac{2\operatorname{tg}\theta}{1-\operatorname{tg}^2\theta} = \frac{1}{2}$$

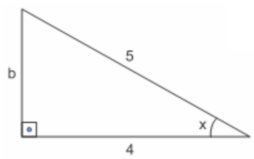
$$\frac{2\cdot\frac{r}{10}}{1-\left(\frac{r}{10}\right)^2} = \frac{1}{2} \Leftrightarrow r^2 + 40r - 100 = 0$$

$$r = (10\sqrt{5} - 20)\ \text{m}.$$

LETRA B

QUESTÃO 02

Se $\cos x = \frac{4}{5}$ e $x \in \left[0, \frac{\pi}{2}\right]$, podemos considerar um triângulo retângulo com um dos ângulos agudos medindo x o cateto adjacente a ele medindo 4 e a hipotenusa medindo



Calculando a medida do cateto b através do Teorema de Pitágoras, podemos escrever:

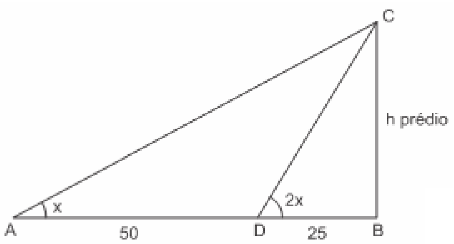
$$b^2 + 4^2 = 5^2 \Rightarrow b = 3.$$

Concluímos então que $\operatorname{tg} x = \frac{3}{4}$ e que:

$$\operatorname{tg}(2x) = \frac{2\cdot\operatorname{tg}x}{1-\operatorname{tg}^2x} = \frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}} = \frac{\frac{3}{2}}{\frac{7}{16}} = \frac{3}{2}\cdot\frac{16}{7} = \frac{24}{7}.$$

LETRA C

QUESTÃO 03



Sendo h a altura do prédio, pode-se escrever:

$$\operatorname{tg}(x) = \frac{h}{75}$$
$$\operatorname{tg}(2x) = \frac{h}{25}$$
$$\operatorname{tg}(2x) = \frac{2\cdot\operatorname{tg}(x)}{1-(\operatorname{tg}(x))^2}$$
$$\frac{h}{25} = \frac{2\cdot\frac{h}{75}}{1-\left(\frac{h}{75}\right)^2} \rightarrow \frac{1}{25} = \frac{2}{75}\cdot\frac{75^2}{75^2-h^2}$$
$$h^2 = 1875 \rightarrow h = 25\sqrt{3}$$

LETRA D

QUESTÃO 04

Desenvolvendo os quadrados, obtemos

$$f(x) = (\operatorname{sen}x + \cos x)^2 + (\operatorname{sen}x - \cos x)^2$$
$$= \underbrace{\operatorname{sen}^2x + \cos^2x}_{1} + 2\operatorname{sen}x\cos x + \underbrace{\operatorname{sen}^2x + \cos^2x}_{1} - 2\operatorname{sen}x\cos x$$
$$= 2.$$

Portanto, como f é constante, segue que a alternativa B é a que apresenta um possível gráfico de f.

LETRA B

QUESTÃO 05

y = 1

LETRA C

QUESTÃO 06

f(x) = 8.2.senx.cosx = 8sen(2.x)

LETRA D

QUESTÃO 07

$$\operatorname{tg}x = 2 \rightarrow \frac{\operatorname{sen}x}{\cos x} = 2 \rightarrow \operatorname{sen}x = 2.\cos x$$

$$\operatorname{sen}^2x + \cos^2x = 1 \rightarrow \cos x = \frac{1}{\sqrt{5}} \text{ e } \operatorname{sen}x = \frac{2}{\sqrt{5}}$$

$$\cos(2x) = \cos^2x - \operatorname{sen}^2x = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$$

$$\operatorname{sen}(2.x) = 2.\operatorname{sen}x.\cos x = \frac{4}{5}$$

$$\frac{\cos(2x)}{1+\operatorname{sen}(2.x)} = \frac{\frac{-3}{5}}{\frac{9}{5}} = \frac{-3}{9} = \frac{-1}{3}$$

LETRA B

QUESTÃO 08

$\cos\theta = \frac{15}{20} = \frac{3}{4}$

$\cos(2.\theta) = 2.\cos^2\theta - 1$

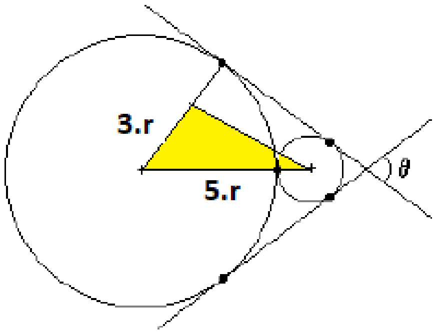
$\cos(2.\theta) = 2.(\frac{3}{4})^2 - 1 = \frac{9}{8} - 1 = \frac{1}{8}$

LETRA E

QUESTÃO 09

No triângulo hachurado a medida da hipotenusa é 5r = 4r + r.

Um dos catetos mede 3.r = 4.r – r e por Pitágoras o outro cateto mede 3.r.



$\text{sen}\left(\frac{\theta}{2}\right) = \frac{3.r}{5.r} = \frac{3}{5}$ e $\cos\left(\frac{\theta}{2}\right) = \frac{4.r}{5.r} = \frac{4}{5}$

$\cos(\theta) = 2.\cos^2\left(\frac{\theta}{2}\right) - 1 = 2.(\frac{4}{5})^2 - 1 = \frac{7}{25}$

LETRA E

QUESTÃO 10

$A = 2.(\text{sena}.\text{cosb} + \text{senb}.\text{cosa})$

$A = 2.\text{sen}(a+b) = 2.\text{sen}\left(\frac{\pi}{6}\right) = 2.\frac{1}{2} = 1$

LETRA D

QUESTÃO 11

$\text{tg}\alpha = \frac{6}{x}$ e $\text{tg}(2.\alpha) = \frac{24}{x}$

$\text{tg}(2.\alpha) = \frac{2.\text{tg}\alpha}{1-\text{tg}^2\alpha}$

$\frac{24}{x} = \frac{\frac{12}{x}}{1-\frac{36}{x^2}} \rightarrow 2 = \frac{1}{1-\frac{36}{x^2}} \rightarrow x = 8,5\text{ m}$

LETRA A

QUESTÃO 12

$\text{sen}(2.\theta) = 2.\text{sen}\theta.\cos\theta$

$2.p = 2.3.p.\cos\theta \rightarrow \cos\theta = \frac{1}{3}$

$\text{sen}^2\theta + \cos^2\theta = 1 \rightarrow \text{sen}\theta = \frac{2.\sqrt{2}}{3}$

$3.p = \frac{2.\sqrt{2}}{3} \rightarrow p = \frac{2.\sqrt{2}}{9}$

LETRA D

QUESTÃO 13

$\text{tg}(2.\theta) = \frac{3}{\sqrt{3}} = \sqrt{3} \rightarrow 2.\theta = 60^0 \text{ e } \theta = 30^0$

$\text{tg}(30^0) = \frac{\sqrt{3}}{3} = \frac{3}{AC} \rightarrow AC = 3.\sqrt{3}$

$CD = AC - AD = 3.\sqrt{3} - \sqrt{3} = 2.\sqrt{3}$

$\text{Área} = \frac{CD.AB}{2} = 3.\sqrt{3}$

LETRA D

QUESTÃO 14

$\text{sen}(60^0 + 45^0) = \text{sen}60^0.\cos45^0 + \cos60^0.\text{sen}45^0$

$\text{sen}(105^0) = \frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos(60^0 - 45^0) = \cos60^0.\cos45^0 + \text{sen}60^0.\text{sen}45^0$

$\cos(15^0) = \frac{\sqrt{6}+\sqrt{2}}{4}$

LETRA A

QUESTÃO 15

$\begin{cases} x + y = 90^0 \\ x - y = 60^0 \end{cases} \rightarrow x = 75^0 \text{ e } y = 15^0$

$\text{senx} + \text{seny} = 2.\text{sen}\left(\frac{x+y}{2}\right).\cos\left(\frac{x-y}{2}\right)$

$\text{sen}75^0 + \text{sen}15^0 = 2.\text{sen}45^0.\cos30^0$

$\text{sen}75^0 + \text{sen}15^0 = 2.\frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{2}$

LETRA C

QUESTÃO 16

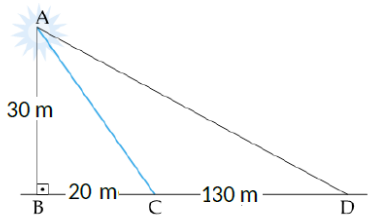
$\text{tg}^2\theta = \frac{9}{16} \rightarrow \text{tg}\theta = \frac{3}{4} \rightarrow \text{sen}\theta = \frac{3}{5} \rightarrow \cos\theta = \frac{3}{5}$

$\text{sen}(2\theta) = 2.\text{sen}\theta.\cos\theta = 2.\frac{3}{5}.\frac{4}{5} = 0,96$

$\frac{\text{sen}\theta}{\text{sen}(2.\theta)} = \frac{0,60}{0,96} = 0,625 = 1 - 0,375$

LETRA D

QUESTÃO 17



$tg(BAC) = \frac{2}{3}$ e $tg(BAD) = 5$

$tg(CAD) = tg(BAD - BAC)$

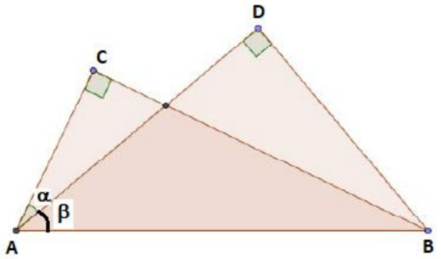
$tg(CAD) = \frac{5-\frac{2}{3}}{1+5.\frac{2}{3}} = \frac{\frac{13}{3}}{\frac{13}{3}} = 1$

Logo, CAD = 45º

LETRA B

QUESTÃO 18

Como AD = BD, então α = 45º.



AC = 1 e BC = 7, temos que tg(α + β) = 7.

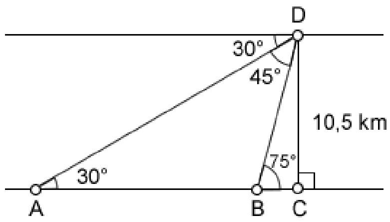
$tg(\alpha + 45^0) = \frac{tg\alpha + tg45^0}{1 - tg\alpha . tg45^0}$

$7 = \frac{tg\alpha + 1}{1 - 1.tg\alpha} \rightarrow tg\alpha = \frac{3}{4} \rightarrow sen\alpha = \frac{3}{5}$

LETRA C

QUESTÃO 19

Vamos traçar uma ortogonal por D.



No triângulo no BCD:

$$\tan 75^\circ = \frac{10,5}{BC} \Rightarrow \tan(30^\circ + 45^\circ) = \frac{10,5}{BC}$$

$$\frac{\tan 30^\circ + \tan 45^\circ}{1 - \tan 30^\circ \cdot \tan 45^\circ} = \frac{10,5}{BC} \Rightarrow$$

$$\frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3}} = \frac{10,5}{BC} \Rightarrow BC = 10,5 \cdot (2 - \sqrt{3})$$

No triângulo no ACD:

$$\tan 30^\circ = \frac{10,5}{AC} \Rightarrow \frac{\sqrt{3}}{3} = \frac{10,5}{AC} \Rightarrow AC = 10,5\sqrt{3}$$

$$AB = AC - BC = 10,5\sqrt{3} - 10,5 \cdot (2 - \sqrt{3}) = 21 \cdot (\sqrt{3} - 1)$$

LETRA A

QUESTÃO 20

Elevando-se as duas expressões ao quadrado:

$$\begin{cases} \cos^2 a + 2.\cos a.\cos b + \cos^2 b = \frac{3}{2} \\ \sin^2 a + 2.\sin a.\sin b + \sin^2 b = \frac{1}{2} \end{cases}$$

Adicionando membro a membro:

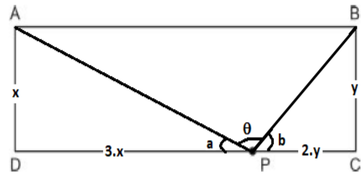
$1 + 2.(\cos a.\cos b + \sin a.\sin b) + 1 = 2$

$2.(\cos a.\cos b + \sin a.\sin b) = 0$

$\cos(a - b) = 0$

LETRA A

QUESTÃO 21



$tga = \frac{x}{3.x} = \frac{1}{3}$ e $tgb = \frac{y}{2.y} = \frac{1}{2}$

$tg(a + b) = \frac{tga + tgb}{1 - tga.tgb} = \frac{\frac{1}{3}+\frac{1}{2}}{1 - \frac{1}{3}\frac{1}{2}} = 1$

$a + b = 45^0 \rightarrow \theta = 135^0$

LETRA A

QUESTÃO 22

Para o triângulo ACD:

$tg\alpha = \frac{CD}{AC} \rightarrow \frac{CD}{17} = \frac{7}{17} \rightarrow CD = 7$

Para o triângulo BCE:

$\beta = 180 - (180 - 45 - \alpha)$

$\beta = 45 + \alpha$

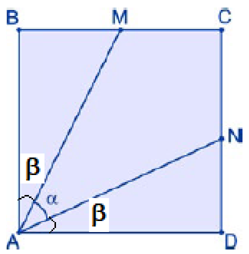
$tg\beta = tg(45^o + \alpha)$

$tg\beta = \frac{tg(45^o) + tg\alpha}{1 - tg(45^o).tg\alpha} = \frac{1 + \frac{7}{17}}{1 - 1.\frac{7}{17}} = \frac{\frac{24}{17}}{\frac{10}{17}} = 2.4$

$tg\beta = \frac{CE}{BC} \rightarrow \frac{CE}{5} = 2,4 \rightarrow CE = 12$

LETRA A

QUESTÃO 23



$\text{sen}\beta = \frac{1}{\sqrt{5}}$ e $\cos\beta = \frac{2}{\sqrt{5}}$

$\text{sen}(2.\beta) = 2.\text{sen}\beta.\cos\beta = \frac{4}{5}$

$\cos(2.\beta) = \cos^2\beta - \text{sen}^2\beta = \frac{3}{5}$

$\alpha + 2.\beta = 90^o \rightarrow \text{sen}(\alpha + 2.\beta) = \text{sen}90^o = 1$

$\text{sen}\alpha.\cos(2.\beta) + \cos\alpha.\text{sen}(2.\beta) = 1$

$\text{sen}\alpha.\frac{3}{5} + \cos\alpha.\frac{4}{5} = 1$

$3.\text{sen}\alpha + 4.\cos\alpha = 5$

Isolando cosα

$\cos\alpha = \frac{5-3.\text{sen}\alpha}{4}$

Combinando com:

$\text{sen}^2\alpha + \cos^2\alpha = 1$

$25.\text{sen}^2\alpha - 30.\text{sen}\alpha + 9 = 0$

$\text{sen}\alpha = \frac{3}{5}$

LETRA A

QUESTÃO 24

$\text{sen}(2x) = \frac{h}{160}$ e $\text{sen}(4x) = \frac{h}{100}$

$\text{sen}(4x) = 2.\text{sex}(2x).\cos(2x)$

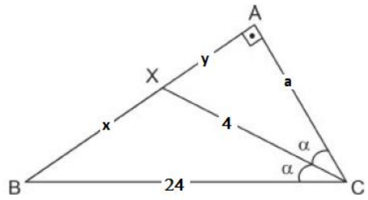
$\frac{h}{100} = 2.\frac{h}{160}.\cos(2x) \rightarrow \cos(2x) = \frac{4}{5}$

$\cos(2x) = \frac{4}{5} \rightarrow \text{sen}(2x) = \frac{3}{5}$

$\text{sen}(2x) = \frac{3}{5} = \frac{h}{160} \rightarrow h = 96\text{ m}$

LETRA C

QUESTÃO 25



$a^2 = 16 - y^2$

$\frac{y}{a} = \frac{x}{24} \rightarrow y = \frac{a.x}{24}$

$a^2 = 576 + (x+y)^2$

a = 3 cm

LETRA B

QUESTÃO 26

$\alpha = C\hat{A}B \rightarrow \text{tg }\alpha = \frac{3r}{3r} = 1$

$\beta = P\hat{A}B \rightarrow \text{tg }\beta = \frac{r}{4r} = \frac{1}{4}$

$\theta = \alpha - \beta \rightarrow \text{tg }\theta = \text{tg }(\alpha - \beta) = \frac{1 - \frac{1}{4}}{1 + 1 \cdot \frac{1}{4}}$

$\text{tg }\theta = \frac{3}{4} \cdot \frac{4}{5} \rightarrow \text{tg }\theta = \frac{3}{5} = 0,6$

LETRA B

QUESTÃO 27

$A = \frac{12 \cdot 16 \cdot \text{sen }60°}{2} = 48\sqrt{3}\ \text{m}^2$

LETRA E

QUESTÃO 28

Sabendo que $\text{sen }70° = \cos 20°$ e $\cos 50° = \text{sen }40°$.
Podemos escrever que:

$A = \frac{2 \cdot \text{sen }20° \cdot \text{sen }70°}{3 \cdot \cos 50° + \text{sen }40°} = \frac{2 \cdot \text{sen }20° \cdot \cos 20°}{3 \cdot \text{sen }40° + \text{sen}40°}$

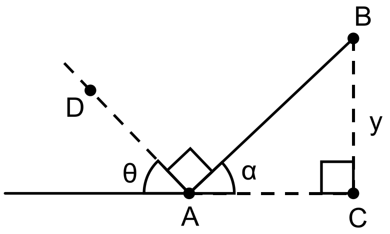
$A = \frac{\text{sen }40°}{4 \cdot \text{sen}40°} = \frac{1}{4}$

Portanto, $\log A = \log\frac{1}{4} = \log 2^{-2} = -2 \cdot 0,3 = -0,6.$

LETRA B

QUESTÃO 29

Considere a figura.



Dado $D\hat{A}B = 90°$, temos $\alpha = 90° - \theta$. Além disso, do triângulo retângulo ABC, vem $\text{sen}\,\alpha = \frac{\overline{BC}}{\overline{AB}} \Leftrightarrow y = 2\,\text{sen}\,\alpha$.

Mas $\text{sen}\,\alpha = \text{sen}(90° - \theta) = \cos\theta$, então, $y = 2\cos\theta$.

LETRA **D**

QUESTÃO 30

Considere a figura, em que h é a diferença pedida.

Sabendo que $\cos 30° = \frac{\sqrt{3}}{2}$, vem

$$\text{sen}^2\left(\frac{30°}{2}\right) = \frac{1 - \cos 30°}{2} \Leftrightarrow \text{sen}^2\,15° = \frac{1 - \frac{\sqrt{3}}{2}}{2}$$

$$\text{sen}\,15° \cong \frac{\sqrt{2 - 1,73}}{2} \Rightarrow \text{sen}\,15° \cong \frac{1}{2} \cdot \sqrt{\frac{27}{100}}$$

$$\text{sen}\,15° \cong \frac{1}{2} \cdot \frac{3 \cdot 1,73}{10} \Rightarrow \text{sen}\,15° \cong 0,26.$$

Portanto, $h = 100 \cdot \text{sen}\,15° \cong 100 \cdot 0,26 = 26\ \text{m}$.

LETRA **B**